

# Circles



## Getting the Idea

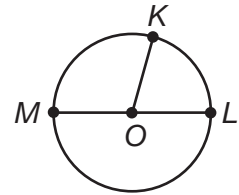
A **circle** is the set of all points in a plane that are the same distance from a given point called the center. A circle is named by its center.

A **radius** is the distance from the center of a circle to any point on the circle.  $\overline{OK}$ ,  $\overline{OL}$ , and  $\overline{OM}$  are radii of circle  $O$ .

A **diameter** is the distance across a circle through its center.

The length of a diameter is always 2 times the length of a radius.

$\overline{LM}$  is a diameter of circle  $O$ .



**Circumference** is the distance around a circle. The circumference of a circle is the product of its diameter and  $\pi$ , or  $\pi d$ . Use 3.14 or  $\frac{22}{7}$  as approximations for  $\pi$  in computations.

The table below shows the formulas for finding the circumference and area of a circle.

Formulas	
circumference	$C = \pi d$ or $C = 2\pi r$
area	$A = \pi r^2$

## Example 1

What is the circumference of this circle? Use 3.14 for  $\pi$ .

**Strategy** Use the formula for the circumference of a circle.

### Step 1

Write the formula for circumference when you know the radius.

$$C = 2\pi r$$

### Step 2

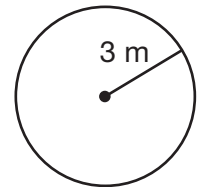
Substitute 3 for  $r$  and 3.14 for  $\pi$ . Then multiply.

$$C = 2\pi r$$

$$C \approx 2 \times 3.14 \times 3 \quad \text{Note: Use } \approx \text{ because 3.14 is an estimate.}$$

$$C \approx 18.84$$

**Solution** The circumference of the circle is about 18.84 meters.



## Example 2

The circumference of a circle is  $9\pi$  inches. What is the diameter of the circle?

**Strategy** Use the formula for the circumference of a circle.

**Step 1**

Write the formula for the circumference when you know the diameter.

$$C = \pi d$$

**Step 2**

Substitute  $9\pi$  for  $C$ .

$$9\pi = \pi d$$

**Step 3**

Divide both sides of the equation by  $\pi$ .

$$9\pi = \pi d$$

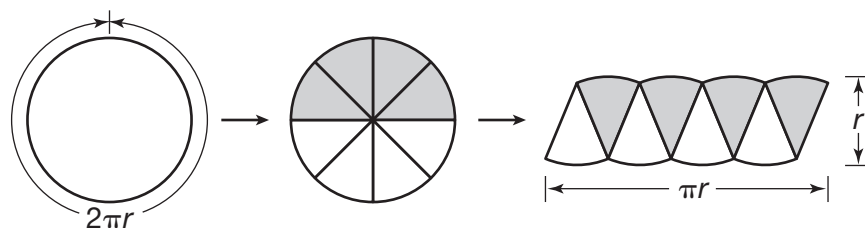
$$\frac{9\pi}{\pi} = \frac{\pi d}{\pi}$$

$$9 = d$$

**Solution** The diameter of the circle is 9 inches.

The formulas for the circumference of a circle and the area of a rectangle can help you find a formula for the area of a circle.

Imagine cutting a circle into an equal number of pieces, such as 8 pieces. Arrange the pieces to form as close to a rectangle as possible.



As you can see, the sides are not straight, so it is not a rectangle. However, as the pieces of the circle get smaller, when arranged to make a rectangle, the sides will be straight.

Since the circumference of a circle is  $2\pi r$ , the length of the rectangle is  $\frac{1}{2}$  the circumference. So, the length is  $\frac{1}{2}C = \frac{1}{2} \times 2\pi r$ , or  $\pi r$ .

The width of the rectangle is about the same as the length of the radius,  $r$ , of the circle. The area of a rectangle is  $lw$ , so the area of the circle is  $\pi r \times r$ , or  $A = \pi r^2$ .

### Example 3

A circle has a diameter of 8 inches. What is the area of the circle? Use 3.14 for  $\pi$ .

**Strategy** Use the formula for the area of a circle.

**Step 1** Use the diameter to find the radius.

The length of the radius is  $\frac{1}{2}$  the length of the diameter.

The diameter is 8.

$8 \div 2 = 4$ , so the radius is 4 inches.

**Step 2** Write the formula for the area of a circle.

$$A = \pi r^2$$

**Step 3** Substitute 4 for  $r$  and 3.14 for  $\pi$ . Solve.

$$A = \pi r^2$$

$A \approx 3.14 \times 4 \text{ in.} \times 4 \text{ in.}$  Again, use  $\approx$  because 3.14 is an estimate.

$$A \approx 50.24 \text{ in.}^2$$

**Solution** The area of the circle is about **50.24 in.<sup>2</sup>**

### Example 4

The area of a circle is  $25\pi$  square centimeters. What is the radius of the circle?

**Strategy** Use the formula for the area of a circle.

**Step 1** Write the formula for the area of a circle.

$$A = \pi r^2$$

**Step 2** Substitute  $25\pi$  for  $A$ .

$$25\pi = \pi r^2$$

**Step 3** Divide both sides of the equation by  $\pi$ .

$$25\pi = \pi r^2$$

$$\frac{25\pi}{\pi} = \frac{\pi r^2}{\pi}$$

$$25 = r^2$$

**Step 4** Take the square root of both sides of the equation to find the value of  $r$ .

$$25 = r^2$$

$$\sqrt{25} = \sqrt{r^2}$$

$$5 = r$$

**Solution** When the area of a circle is  $25\pi$  square centimeters, the radius is **5 centimeters**.



## Coached Example

Philip is building a go-cart. The wheels he uses on the go-cart have a radius of 6 inches. What are the circumference and the area of each wheel?

What is the formula for the circumference of a circle when the radius is given?

\_\_\_\_\_

Use 3.14 for  $\pi$  and substitute the length of the \_\_\_\_\_ into the formula.

$$C \approx \underline{\hspace{2cm}}$$

Multiply.

$$C \approx \underline{\hspace{2cm}}$$

What is the formula for the area of a circle? \_\_\_\_\_

Use 3.14 for  $\pi$  and substitute the length of the \_\_\_\_\_ into the formula.

$$A \approx \underline{\hspace{2cm}}$$

Multiply.

$$A \approx \underline{\hspace{2cm}}$$

**The circumference of each wheel is about \_\_\_\_\_,**  
**and the area is about \_\_\_\_\_.**



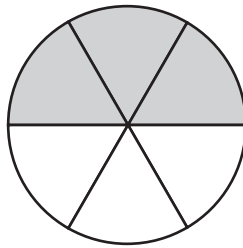
## Lesson Practice

Choose the correct answer.

- A rose garden is circular. The diameter of the garden is 18 feet. Which is closest to the total area of the garden? Use 3.14 for  $\pi$ .
  - 56.52 ft<sup>2</sup>
  - 63.59 ft<sup>2</sup>
  - 113.04 ft<sup>2</sup>
  - 254.34 ft<sup>2</sup>
- A circular swimming pool has a radius of 15 feet. The family that owns the pool wants to put up a circular fence that is 5 feet away from the pool at all points. Which is closest to the circumference of the fence they will need? Use 3.14 for  $\pi$ .
  - 94.2 ft
  - 125.6 ft
  - 157 ft
  - 188.4 ft
- Lana is putting lace trim around the border of a circular tablecloth. The tablecloth has a diameter of 1.2 meters. To the nearest meter, what is the least amount of lace she needs? Use 3.14 for  $\pi$ .
  - 3 m
  - 4 m
  - 7 m
  - 8 m
- Randy's bicycle tires have a diameter of 42 centimeters. Which is closest to the circumference of one of the tires? Use 3.14 for  $\pi$ .
  - 65.94 cm
  - 87.14 cm
  - 131.88 cm
  - 441 cm
- Pete needs to install a circular window with a radius of 7.25 inches. Which is closest to the amount of glass he will need? Use 3.14 for  $\pi$ .
  - 22.77 in.<sup>2</sup>
  - 41.26 in.<sup>2</sup>
  - 45.53 in.<sup>2</sup>
  - 165.05 in.<sup>2</sup>
- At Palermo Pizzeria pizzas are sold by their diameter. Rihanna orders a 14-inch pizza. Which is closest to the area of the pizza? Use 3.14 for  $\pi$ .
  - 21.98 in.<sup>2</sup>
  - 43.96 in.<sup>2</sup>
  - 153.86 in.<sup>2</sup>
  - 307.72 in.<sup>2</sup>

7. A circular pin has a diameter of 6.2 centimeters. Which is closest to the area of the pin? Use 3.14 for  $\pi$ .
- A.  $30.18 \text{ cm}^2$   
 B.  $19.47 \text{ cm}^2$   
 C.  $15.54 \text{ cm}^2$   
 D.  $9.74 \text{ cm}^2$
8. What is the radius of a circle when the circumference is  $16\pi \text{ cm}$ ?
- A. 16 cm  
 B. 12 cm  
 C. 8 cm  
 D. 4 cm

9. The circle below is divided into 6 equal pieces.



- A. Explain how to use the formulas for the circumference of a circle and the area of a rectangle to find the formula for the area of a circle.

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- B. Use your explanation from Part A to find the area of the circle if the length of the radius is 3 centimeters. Check that the area is the same when you use the formula for the area of a circle. Show your work.

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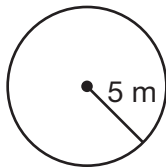
10. Tires come in several sizes. The radii of three tires are given. Circle the approximate circumference of each tire. Use 3.14 for  $\pi$ .

13 in.:	16.14	in.	14 in.:	43.96	in.	16 in.:	50.24	in.
	40.82			87.92			100.48	
	81.64			615.44			803.84	

11. Draw a line from each area of a circle in the left column to the corresponding circumference of the circle in the right column.

- |                                  |                 |
|----------------------------------|-----------------|
| A. $25\pi$ units <sup>2</sup> ●  | ● $4\pi$ units  |
| B. $16\pi$ units <sup>2</sup> ●  | ● $20\pi$ units |
| C. $100\pi$ units <sup>2</sup> ● | ● $10\pi$ units |
| D. $4\pi$ units <sup>2</sup> ●   | ● $8\pi$ units  |

12. Look at the circle. Which is a true statement? Circle all that apply.



- A. The diameter is 5 m.
- B. The circumference is  $5\pi$  m.
- C. The diameter is 10 m.
- D. The area is  $25\pi$  m<sup>2</sup>.
- E. The circumference is  $25\pi$  m.
- F. The radius is 5 m.
- G. The area is  $10\pi$  m<sup>2</sup>.

13. A circular flower garden has an area of  $81\pi$  square inches. Select True or False for each statement.

- A. The circumference is  $9\pi$  in.       True    False  
 B. The radius is 9 in.                 True    False  
 C. The diameter is 9 in.               True    False  
 D. The circumference is  $18\pi$  in.    True    False

14. Dartboards come in different sizes. Circle the approximate area of a dartboard given each diameter. Use 3.14 for  $\pi$ .

45.1 cm:	142	$\text{cm}^2$	25 cm:	79	$\text{cm}^2$	45.7 cm:	143	$\text{cm}^2$
	1,597			491			1,639	
	6,387			1,963			6,558	

15. The bottom of a cup is a circle with a diameter of  $4\frac{1}{4}$  in. Which is a true statement? Circle all that apply. Use 3.14 for  $\pi$ .

- A. The radius is  $8\frac{1}{2}$  in.  
 B. The circumference is about 13.3 in.  
 C. The area is about  $14.2 \text{ in}^2$ .  
 D. The area is about  $56.7 \text{ in}^2$ .  
 E. The circumference is about 26.7 in.  
 F. The radius is  $2\frac{1}{8}$  in.
16. A small swimming pool has a circumference of  $3\pi$  feet. Is each statement about the pool true? Select Yes or No.
- A. The radius is 3 ft.                 Yes    No  
 B. The diameter is 3 ft.               Yes    No  
 C. The diameter is 1.5 ft.            Yes    No  
 D. The radius is 6 ft.                 Yes    No